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INVESTIGATING LEARNERS' CONCEPTUAL AND PROCEDURAL UNDERSTANDING OF FRACTIONS IN GRADE 6 MATHEMATICS CURRICULUM

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ABSTRACT

The study investigated Grade 6 learners' conceptual and procedural understanding of fractions in primary school Mathematics. The objective of the study was to investigate the challenges experienced by primary school learners in dealing with fractions. The study was guided by quantitative methodology, framed from the positivist paradigm using an explorative case study design. One primary school was randomly selected in the Manzini region of Eswatini. The randomisation process came up with a peri-urban school which was single streamed with 48 learners. A written diagnostic test was administered to generate data where the findings were presented using descriptive analysis techniques. The data was analysed using the concept evaluation scheme that classified learners' responses. Findings revealed that a few learners had limited mathematical understanding of concepts related to fractions, where learners carried out mathematical procedures without conceptual understanding. Learners were found to be committing errors due to computational weaknesses and incomplete mastery of the number system. Findings further reveal that learners were committing conceptual and procedural errors which posed specific misconceptions and even limited understanding in application of operations. In problem solving which is a higher application of fractions, learners showed no conceptual and procedural understanding of fractions. The study recommends that teachers develop practices and lessons that begin by exposing learners' perspectives about fractions and consistently exploring fractions such as one-half, one-fourth and one-tenth. Teachers should increase conceptual development of fraction units, fraction relationships and fraction operations. Finally, teachers should assist learners to make real-life connections with fractions so that the learners can conceptualize fractional word problems.

KEYWORDS: fractions, conceptual understanding and procedural understanding

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INTRODUCTION

A fraction is a numerical quantity that is part of a whole. It represents a part of a whole or, more generally, any number of equal parts (Hurrell & Day, 2014). Page (2018) points out that, a fraction is a number between zero and one and is expressed as one number over another; the number at the bottom being a denominator and the number on top being a numerator. In Mathematics, fractions are involved in probabilistic, proportional and algebraic reasoning (Gabriel, 2013). Gabriel (2013) further states that fractions are also part of human everyday life, fractions have been used for centuries and are manipulated in a variety of everyday life situations such as, estimation, following a recipe, reading a map and many more applications.

Despite the fact that fractions apply to everyday life situations, learners still have difficulty in understanding fractions (Gabriel, 2013). According to Lamon (2012), the various fraction operations such as addition, subtraction, multiplication, division, comparing and simplifying can be difficult for most learners. Lamon (2012) further explains that the lack of conceptual understanding is one of the many reasons learners face challenges when solving problems in fractions. It is further explained that learners tend to memorize the procedures involved in fraction operations without conceptual understanding. Consequently, they fail to connect with anything about the operations, but instead, they just follow algorithm.

Background of the study

Fractions are important in learning Mathematics and are building blocks for other Mathematics skills (Hannich, 2009). Limited understanding of fractions, interferes with learner's ability to learn algebra. Working with fractions introduces some of the essentials of the number theory, such as the lowest common denominator, greatest common factor and prime factorization. Siegler (2010) further acknowledge that most learners are challenged by fraction arithmetic, where the four Mathematics operations present difficulty and learners often confuse the operations. The section discusses what is expected from primary learners concerning fractions, by exploring the Eswatini Primary Certificate Structural Model, Eswatini Primary Certificate Mathematics Syllabus and the curriculum content of fractions. In the kingdom of Eswatini, the Eswatini Primary Certificate (EPC) syllabus is designed as a seven-year course for examination in Grade 7, where the learners sit for two Mathematics papers; Paper 1, which is a compulsory short-answer paper and Paper 2, which is a compulsory structured or long answer paper. The examination is a locally based assessment administered by the Examinations Council of Eswatini (ECESWA) through schools. ECESWA is a quasi-government organization with the mandate to administer examinations and issue certificates to primary, junior secondary and senior secondary school graduates in Eswatini. They also analyse data, produce and present reports and process results. It was established through the Education Act No. 9 of 1981 and its mission is to provide efficient examination management and administration services (The Government of Eswatini, 2018). The ECESWA examiners' reports over the years in Mathematics have reported that most learners face challenges in conceptual understanding, interpreting and solving problems in the topic; fractions (ECESWA, 2019).

The syllabus assumes that learners have acquired knowledge, understanding and skills in their everyday life activities at home and in the community. The curriculum content of the syllabus is arranged into topics covering four areas: Number & Measurement; Shape, Position and Space; Information Handling and Problem Solving but it is treated throughout in a holistic approach. It is intended to promote imaginative and innovative styles of teaching and learning so that the course is enjoyable to all learners, and is designed to assess what learners know, understand and can do. As such, it forms the basis for the development of fundamental tools for learners to progress to higher-level courses of mathematical studies. The syllabus also acts as an instrument that directs instruction and assessment in the classroom, as well as guide examinations. It is structured such that it aligns with the Junior Certificate (JC) (ECESWA, 2018). The curriculum content of the syllabus as arranged by ECESWA (2018) is divided into topics covering four areas where the concepts of fractions are taught: Number and Measurement; Shape, Position and Space; Information Handling and Problem Solving.

(1) Number and Measurement

Number is the foundation of Mathematics. Number enables learners to interpret and represent the world in which they live in. In this topic, learners should develop understanding of concept of number and competencies in using mental and written strategies for solving problems. Learning to recognise, analyse, describe and represent patterns and number relationships connects Mathematics to the world and helps the learner to appreciate fully the value of such pleasures as art, science and other disciplines. Being able to see and use patterns has been identified as a fundamental skill needed for developing mathematical understanding. Algebra serves as a bridge between arithmetic and more broadly generalise mathematical situations. These generalisations can be expressed in words, tables and charts. In later years, learners use the notation of formulas and graphs to represent these generalisations. Hands-on, interactive investigations, using non-standard and standard units, help learners develop an understanding of the many measurable attributes of physical objects. Measurement including length, time, temperature, capacity, weight, mass area, volume, and angle will benefit from this approach. This approach helps build an accessible measurement vocabulary and a meaningful comprehension of what it means to measure.

(2) Information handling

Numeracy and literacy learning are linked by Statistics and Probability. Numbers, logical reasoning and texts interweave to describe phenomena visually, numerically and verbally in what is termed data. Reading and recording data is very important in our daily lives. We learn about the power of evidence as we develop the skills to make statements and evaluate arguments based on data. We learn the power of the question and the framer of the question when we collect and represent data, and we learn that sometimes true or false, pictures are created when we express data into statistics. Data is a powerful descriptive tool.

(3) Shape, Position, and Space

Geometry helps learners represent in an orderly fashion what they see in the world.

In Geometry, learners learn about the features, properties and representation of two-dimensional shapes and three-dimensional objects.

(4) Problem Solving

Word problems which reflect the social life and related to the environment of the child are of particular importance. It is the primary goal of all Mathematics instruction and an integral part of all mathematical activity. In problem solving, learners learn to solve routine and non-routine problems using problem solving steps. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned (ECESWA, 2018).

Generally, learning of fraction involves manipulation of the fraction operations (Siegler, 2010). According to the site Homeschoolmath.net (2015), the general content to be covered in topic fractions which are relevant at primary level worldwide are presented in the Table 1 below; these are the different operations learners need to understand when dealing with fractions. At grade 6 level, the curriculum content in fractions is presented in Table 1 below as a level of focus in this study:

Table 1: Curriculum content of fractions

Topic	Objectives	Content
Fractions	1. Change common fractions to decimal fractions	1. Common fractions to decimal fractions
	1. Order common and decimal fractions by size.	2. Ordering common and decimal fractions
	2. a) round to the nearest whole number b) round off decimal fractions to the nearest tenths and hundredths	3. a) round off the nearest whole number b) found off decimal fractions
	3. add and subtract common fractions with different denominators using LCM	4. Add and subtract common fraction
	4. multiplying a unit fraction	5. Multiplying a unit fraction by a whole number
	5. Multiplying a common fraction by a whole number.	6. Multiplying a common fraction by a whole number.
	6. Add and subtract mixed numbers.	7. Addition and subtraction of mixed numbers.
	7. Express common fractions with denominators that are factors of 100 as percentages.	8. Changing common fractions to percentage.
	8. Change mixed numbers to improper/top-heavy fraction or vice versa.	9. Changing mixed numbers to top-heavy fractions.

REVIEW OF RELATED LITERATURE

This section presents a systematic review of related studies about conceptual and procedural understanding of fractions, misconceptions and errors in fractions, sources of misconceptions and errors in the topic of fractions. The reviewed literature has been sourced from research reports; commentary notes on research work on the teaching and learning of fractions. Learning fractions requires both conceptual and procedural understanding (Arslan, 2010). Conceptual understanding is knowing more than isolated facts and methods.

According to Arslan (2010), conceptual learning involves understanding and interpreting concepts and the relations between concepts. Clarke, Roche and Mitchell (2008) states that, understanding fractions means understanding all the possible concepts that fractions can present. Procedural understanding, on the other hand, is learning that involves only memorizing algorithm (Arslan, 2010).

Clarke, Roche and Mitchell (2008) argue that, there is a gap between conceptual and procedural understanding of fractions among learners. This is evident when the learners fail to recognize different representations of fractions and to use the correct algorithm. Siegler (2010) states that, learners need to have a basic conceptual understanding of fractions before the procedural understanding. From this basic understanding, learners may then begin the arithmetic part of the fractions; understanding the underlying concepts (Sarangarm, 2018).When learners come to school, they already have some basic understanding of fractions; however, most learners carry misconceptions that lead to errors in computing fractions (Siegler, 2010). A misconception is a view or opinion that is incorrect because it is based on faulty thinking or understanding. Errors, on the other hand, are mistakes made by learners as a result of carelessness, misinterpretation of symbols and texts, lack of relevant experience or knowledge related to a mathematical topic, learning objective or concept, lack of awareness or inability to check the answer given (Hansen, 2006).

Battista (2001) states that, the way in which learners construct knowledge is dependent on the cognitive structures' learners have previously developed. This means that there are conceptions and preconceptions that learners of different ages and backgrounds bring with them to the classroom, and these preconceptions may be misconceptions. The preconceived knowledge that learners bring to class may be faulty and not consistent with regular fractional concepts before the learning process takes place in the classroom. Another source of misconception comes from the teacher. According to Siegler (2010), teachers' understanding of fractions can be relatively shallow, as a result, failing to give legitimate explanations of how procedures work or use wrong examples to explain a concept. This is crucial because it can be transferred to the learner. Language can also be a source of misconception.

According to Kaur (2004), the language used to describe fractions can be problematic for some learners. For example, Muira (2001) explains that, a fraction expressed as three over four maybe confusing on what actions to take in terms of the fact that fractions have three distinct meanings. In addition, misconceptions in fractions also arise from the fact that fractions are not natural numbers yet primary school learners spend most of their time learning natural numbers. This is crucial because by the time they are taught fractions, they would have developed schema about how natural numbers work, which is contrary to how fractions work. Some learners will then try to fit what they learn in fractions to their existing knowledge on natural numbers by assuming that the numbers all work the same way; which is simply not the case (Lamon, 2012).

Conceptual framework.

The study is guided by the concept-evaluation scheme of Muchtar (2012). Muchtar (2012) conceptual framework is an evaluative tool that gives an insight and understanding into the relationship that exist between the curriculum and society. The concept-evaluation scheme developed by Muchtar (2012) was adopted and adapted to analyse the data in this study. This tool ranks concepts to determine their relative merits. Contextualised to the study, concept-evaluation technique is useful to evaluate learners' understanding of fraction concepts. Muchtar (2012) classifies learners' responses to a test into seven distinct categories. These categories are presented in the Table 2 below:

Table 2: The seven analysis categories

Degree of understanding	Criteria for scoring
Mathematically Correct (MC)	These are Mathematically complete responses and correct explanations. In relation to this study, it was a learner who was able to follow algorithm by showing all relevant methods with correct answer and reason with understanding if necessary.
Partial Correct (PC)	Mathematically correct responses but incomplete explanations. These were learners who were writing answers only without the working or give incomplete explanations.
Specific Misconception (SM)	Completely Mathematically unacceptable responses or explanations.
Conceptual Error (CE)	These were learners who applied wrong concepts to questions.
Procedural Error (PE)	These were learners who applied the correct concepts but committed computational errors and made mistakes.
No Understanding (NU)	Learners who made irrelevant or unclear responses.
No Response (NR)	These were learners who did not attempt the questions

Key: *Mathematically Correct (MC), Partial Correct (PC), Specific Misconception (SM), Conceptual Error (CE), Procedural Error (PE), No Understanding (NU), No Response (NR): adapted from Muchtar (2012) classification.*

Statement of the problem

Fractions are one of the most important topics learners need to understand in order to be successful in most branches of Mathematics such as algebra, yet it is also an area in which most learners find difficult. According to ECESWA (2018) report, in the Eswatini Primary Certificate on Mathematics, most learners were challenged by questions that contained fractions. Examiners' report - Mathematics Paper 2 (2018): it was reported that many pupils were challenged by question 2 which required the learners to use the number line to add and subtract common fractions with the same denominator, despite the example that was given to them. It is reported that when computing the sum of $^2/_5$ + $^4/_5$, some pupils added both the numerator and the denominator, to get $^6/_{10}$ as their answer. In another addition problem, learners were given a blank number line and they were expected to label before showing $^5/_7$ - $^3/_7$. It is further reported that few candidates answered that question correctly. A majority of learners failed to label correctly the number line using sevenths and also failed to find the difference of $^5/_7$ - $^3/_7$. Furthermore, candidates were also challenged by questions on fractions contained in the short-answer paper: For example, questions 5 and 9 in Section A and question 16 (c).

Section B were questions on fractions which required candidates to perform certain operations. In question 5, candidates were asked to write $^4/_{20}$ in its simplest form, while question 9, required candidates to add $^2/_3$ and $^1/_4$. In question 16 (c), they were expected to demonstrate their knowledge of dividing a whole number by a unit fraction. These questions proved challenging to a majority of the candidates. ECESWA (2017) report, stated that candidates could not even attempt word problems on fractions in Paper 2 and those who attempted could not come with appropriate method of calculating and their answers were not accurate. General comments included: confusing dividing a whole number by a unit fraction with multiplying a whole number by a unit fraction, example, $24 \div {}^{1}/_{3}$ to $24 \times {}^{1}/_{3} = {}^{24}/_{3}$ and expected answer was 8. These examples proved that candidates were challenged by the topic of fractions in Mathematics. On the basis of the examiner's narrative report of 2018 and other preceding reports over the years demonstrate learners' difficulties in handling the topic of fractions. This study therefore seeks to investigate learners' conceptual and procedural understanding of fractions.

The highlighted challenges show that understanding fractions is a challenging area of Mathematics for learners. This is crucial because the challenges and misunderstandings learners face in understanding fractions persist into adult life and pose problems in such wide-range of fields such as medicine and health care, construction and computer programming. The fields of Science, Technology, Engineering and Mathematics (STEM) demand considerable base in fraction knowledge; consequently, a shaky ground in fractions can prevent an individual from pursuing advanced Mathematics and limit learners from several career opportunities later in their life (Outhred & Michelmore, 2006).

Objectives and Research questions

The objective of the study was to investigate learners' conceptual and procedural understanding of fractions in primary school Mathematics curriculum. The objective of the study was to:

- investigate the challenges experienced by primary school learners at grade 6 level on the topic of fractions.

The study was designed to answer the following research question:

- What are the challenges experienced by primary school learners at grade 6 level on the topic of fractions?

METHODOLOGY

Research paradigm

A research paradigm is a belief system (or theory) that guides researchers in their endeavour to interpret the world around, or more formally establish a set of practices (Cohen et al., 2011; Creswell, 2009, Teddlie & tashakkori, 2013). In this research study a positivist paradigm was adopted as an investigative approach to understand learners' conceptual and procedural constructs in learning of fractions in Mathematics curriculum.

Research approach

This study used a quantitative research approach. Creswell (2013) viewed quantitative research approach as a plan and procedure that consists of the steps of broad assumptions to detailed method of data collection, analysis and interpretation. It is based on the nature of the research problem being addressed (Chetty, 2016).

The study adopted the qualitative research approach since it is an inquiry-based approach useful for exploring and understanding a central phenomenon (Creswell, 2008).

Research design

Betram and Christiansen, (2014), Creswell and Plano, (2007), Rule and John, (2011) viewed in this particular research study, a case study design as being more adaptable and often used by researchers in the interpretive paradigm. A case study approach was more relevant in carrying out this research because of the following justifications: case studies provide a rich and vivid description of events; focuses on particular individual actors or / and their ways of executing tasks and focuses upon particular events within the case (Creswell, 2005). The study employed a case study design since it used a single class of grade 6 learners from the Manzini region of Eswatini.

Population

There are 600 primary schools at Eswatini, constituting government, mission, and private schools. These schools are spread in the four regions and are located in urban, semi-urban and rural areas. The overall enrolment of grade 6 is estimated to be 36000 learners.

Sampling

According to Strydom (2013), sampling process is identifying a small portion of the total set of participants. Sampling of participants centres on the notion and principle of representativeness of the population which means that the characteristics of the participants ought to mirror those of the whole population as proposed by the research topic. This study used randomised sampling technique in selecting the school to participate in the study. One class of grade 6 Mathematics learners in the semi-urban location in the Manzini region was sampled. The school was found to have a single stream with 48 learners. All the 48 learners participated in data collection.

Instrumentation

In this study, a written diagnostic test was used as an instrument for data collection. The test consisted of questions that were designed using the following grade 6 materials: Mathematics pupils' book, past tests, national external examinations papers (2018), and internet materials on the topic fractions. The questions focused on the subtopics that are covered in the fraction topics in the Mathematics curriculum. The instrument that was used to test learners on the concepts and operations of fractions included the concepts of addition, subtraction, multiplication and division. Applying fractions through problem solving, as well as the concept of comparing relative size of fractions, was also tested.

In ensuring the content validity, the test was given to experts: the ECESWA Mathematics subject leader and the researcher's colleagues who are majoring in Mathematics. Reliability of the test was calculated through a split-half procedure to acquire the internal consistency. Reliability is the extent to which the test measurement procedure produces the same results on repeated trials (Oluwatayo, 2012). The Cronbach Alpha was used to calculate internal consistency of the test which was 0.79 while piloting the test.

Data analysis methods

The study adopted quantitative data analysis tools with particular reference to descriptive statistics. The test was graded by the researcher quantitatively guided by Muchtar's (2012) evaluative framework. The scoring rubric provided the marks allocations and the categorisation of all the learners' responses. The data was categorized and tabulated into tables for analysis using the conceptual framework. Grouping the learners' responses into categories helped in identifying learner's competency-based skills, learners' process skills and competences in handling Mathematical knowledge and understanding in the topic fractions. Descriptive analysis was used to analyse the data to understand the types of errors and misconceptions held by the learners, their level of understanding; procedurally and conceptually, their strengths and weaknesses.

PRESENTATION OF RESULTS

Learners' understanding of the concepts were analysed using the concept evaluation scheme, as a method of data analysis, after the tests were marked. Learners' responses were classified into seven different categories, that is, Mathematically Correct (MC), Partial Correct (PC), Specific Misconception (SM), Conceptual Error (CE), Procedural Error (PE), No Understanding (NU) and No Response (NR). Analysis was done for each question based on the fraction concepts that were tested and were also graded and classified based on the seven categories of the study.

Data presentation

The data presented consists of results from the test administered in the school. The descriptive analysis technique was adopted for presenting the data. According to Creswell (2008), this technique is appropriate since it describes the present condition and the nature of a condition as it takes place during the time of the study. The questions in the test were scored, the learners' performance systematically distributed according to the adopted analysis framework. The presentation of data have been tabulated using the structure of the diagnostic test which had four (4) questions.

Table 3: Learners understanding, shading and comparing sizes of fractions

Q.1	*CONCEPTS*	*MC*	*PC*	*SM*	*CE*	*PE*	*NU*	*NR*
A.	Meaning of a fraction	22	18	1	0	0	6	1
		46%	38%	2%	0%	0%	13%	2%
B. a)	Shading a given	47	0	0	0	0	1	0
	fraction	98%	0%	0%	0%	0%	2%	0%
b)		19	0	0	0	0	29	0
		40%	0%	0%	0%	0%	60%	0%
C. a)	Comparing fraction	14	0	0	0	0	29	5
	size	29%	0%	0%	0%	0%	60%	10%
b)		21	0	0	0	0	22	5
		44%	0%	0%	0%	0%	46%	10%

Key: *Mathematically Correct (MC), Partial Correct (PC), Specific Misconception (SM), Conceptual Error (CE), Procedural Error (PE), No Understanding (NU) and No Response (NR)*

Question 1(A), required the learners to reflect their conceptual understanding of the definition of fractions. The results show that 22 (46%) of the learners defined a fraction in terms of the part-whole concept. According to Cramer, Monson, Whitney, Leavitt and Weberg (2010), using the part-whole construct is an effective starting point for building meaning of fractions. While 18(38%) gave the similar response which was categorized as being partially correct (PC). About 1(2%) of the learners gave definitions that conveyed specific misconceptions (SM). For example, defining a fraction as half of something, as a number that is changed to a decimal number as well as defining a fraction as an improper number. The learners were giving definitions that were not connected to fraction concepts and some of these definitions had no meaning at all. Furthermore analysis, 6(13%) and 1(2%) of these learners were categorised as (NU) and (NR) respectively who did not attempt the question.

Question 1, (B) was divided into two parts, (a) and (b). Both questions required the learners to shade a given fraction in the given shapes. The fist shape was a square divided into four parts, and the learners were asked to shade ¾. Only 1(2%) learner was unable to correctly shade the given fraction which showed no understanding (NU), while 47(98%) got it correct. Part (b) required learners to shade ½ in a hexagon divided into six parts. An example below demonstrates the common response, see figure 1 below.

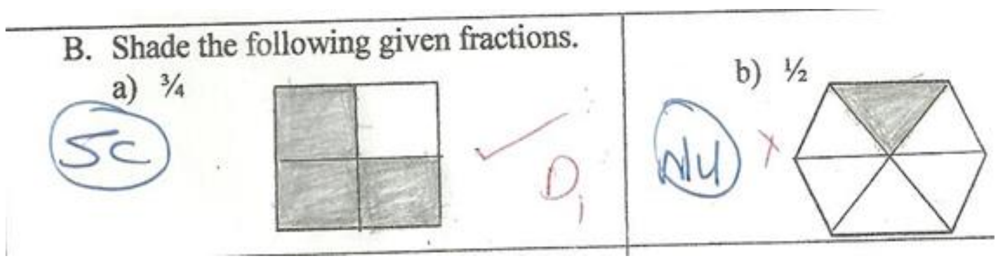

Figure 1: A sample of a response from a learner showing no understanding.

This shows that these learners have no concrete understanding of the part-whole concept, that is, they do not have meaning to the 'number of parts taken out of a whole concept,' and also do not understand what a denominator or numerator stand for. Also, this showed that the learners had no understanding of showing equal-sized parts especially in a shape such as the hexagon which was divided into six equal parts. Proportional representation 29(60%) and 5(10%) of the learners were categorized under (NU) and (NR) respectively in this question, and most of them shaded the numerator only. The other learners 14(29%) shaded correctly.

Question 1, (C) tested the learners' ability to compare fractions in terms of their sizes. Part (a) required learners to write down the larger fraction between $^{1}/_{2}$ and $^{1}/_{3}$. This question was challenging to most learners. Proportionally, there were 29(60%) and 5(10%) who were categorised as (NU) and (NR). This is a result of pupils choosing $^{1}/_{3}$ instead of $^{1}/_{2}$. This error is a result of pupils choosing the fraction with a larger denominator. Cramer and Whitney (2010) explain that, most learners have a misconception of thinking that a fraction such as $^{1}/_{5}$, as example, is smaller than a fraction of $^{1}/_{10}$ because 5 is less than 10. Part (b), learners were to write down the larger fraction between $^{2}/_{5}$ and $^{3}/_{4}$. Again learners presented no understanding to this question.

Proportionally, 22(46%) were categorized under NU while 5(10%) were categorised as NR and the rest of the learners responded correctly 21(44%).

Table 4: Learners competencies, process skills and handling of operations in fractions

Q.2	*CONCEPTS*	*MC*	*PC*	*SM*	*CE*	*PE*	*NU*	*NR*
A. a)	Adding fractions -	7	2	11	20	0	6	0
	same and those	15%	4%	24%	43%	0%	13%	0%
b)	different	2	1	10	25	1	7	0
	denominators.	4%	2%	22%	54%	2%	15%	0%
B. a)	Subtracting fractions	12	1	10	13	1	9	0
	- same and different	26%	2%	22%	28%	2%	20%	0%
b)	denominators.	5	1	10	18	0	12	0
		11%	2%	22%	39%	0%	26%	0%
C.	Concepts of adding	2	3	0	0	0	39	2
	and subtracting	4%	7%	0%	0%	0%	85%	4%
	fractions.							

Key: *Mathematically Correct (MC), Partial Correct (PC), Specific Misconception (SM), Conceptual Error (CE), Procedural Error (PE), No Understanding (NU) and No Response (NR)*

This question generally required learners to apply their acquired skills of addition and subtraction concepts to fractions. Analysis of this question distributed learners' responses to the other categories.

Question 2, (A) was divided into two parts. Part (a), where learners were required to add fractions that had the same denominator. There were 7(15%) of these learners who were conceptually correct, 2(4%) were partially correct since they gave the answer only, without the working, 11(22%) of the learners. Furthermore 11(24%) were categorized under SM. These learners treated fractions as whole numbers and were working on the fraction sum using the whole number concept. For example some pupils answered part (a) as $^{3}/_{6} + ^{2}/_{6} = ^{5}/_{12}$. According to McNamara and Shaughnessy (2010), learners tend to overgeneralize their whole-number knowledge and transfer wrongly which is a misconception. Also, 20(43%) of these learners displayed conceptual errors, while 6(13%) showed no understanding. The figure 2 below demonstrates the specific misconceptions the learners possessed, where learners overgeneralizes the whole number concept such that it overlaps to the fraction concept.

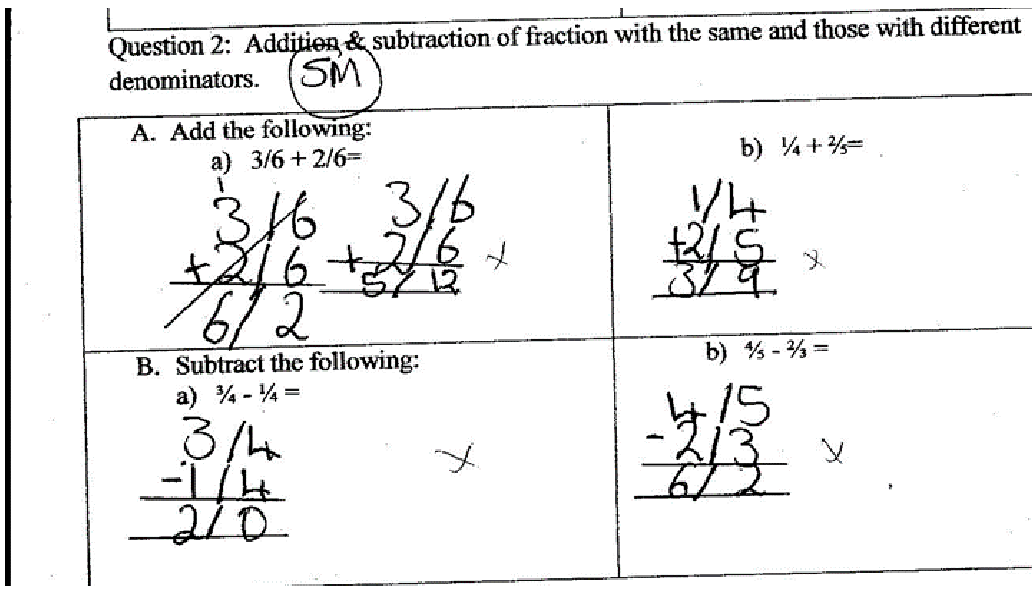

Figure 2: A sample of working from a learner showing specific misconceptions.

In part (b), learners were required to add fractions that have different denominators, that is, ¼ + ⅖. Some learners displayed conceptual errors because they did not show an understanding of the concept of adding common fractions with different denominators. They failed to use the LCM method nor the equivalence approach. Proportional distribution was 5(11%) of MC, 1(2%) of PC, 10(22%) of SM, 18(39%) of CE, and 12(26%). This particular question presented the wider spectrum of learners' deficiencies in the conceptualisation of fractions.

Lastly, **question 2, (C),** required the learners to account for their working in this question by writing down a rule they used for addition and subtraction. This question was meant to find out if learners understood the concepts they were applying or they were just carrying out the procedures without any understanding. The results show that learners were unable to explain the concepts used when adding or subtracting fractions. Only 2(4%) were MC category, 3(7%) were PC while 39(85%) were NU and 2(4%) were NR. The learners had no understanding of what they were doing.

Table 5: Learners competencies, process skills and handling of operations in fractions

Q.3	*CONCEPTS*	*MC*	*PC*	*SM*	*CE*	*PE*	*NU*	*NR*
a)	Multiplication of fractions:	1 2%	7 15%	0 0%	35 73%	1 2%	4 8%	0 0%
b)		0 0%	5 10%	0 0%	25 52%	6 13%	8 17%	4 8%
c)		4 8%	7 15%	0 0%	12 25%	20 42%	4 8%	1 2%
d)	Fraction division	0 0%	1 2%	0 0%	13 27%	11 23%	22 46%	1 2%
e)	Concept of multiplying dividing fractions	0 0%	5 10%	0 0%	0 0%	0 0%	35 73%	8 17%

Key: *Mathematically Correct (MC), Partial Correct (PC), Specific Misconception (SM), Conceptual Error (CE), Procedural Error (PE), No Understanding (NU) and No Response (NR)*

The question required the learners to multiply and divide fractions of similar and different denominators. These concepts were tested in four (4) parts in question 3. The overall results showed that Conceptual Errors (CE) and Procedural Errors (PE) a category of errors dominated this question followed by those who showed No Understanding (NU) and No Response (NR) categories. In part (a) show that 1(2%) and 7(15%) of learners were categorized under MC and PC respectively. These are pupils who were able to carry out all the necessary workings correctly, and also gave correct and reasonable answer. Most of the learners were adding instead of multiplying demonstrated through figure 3.

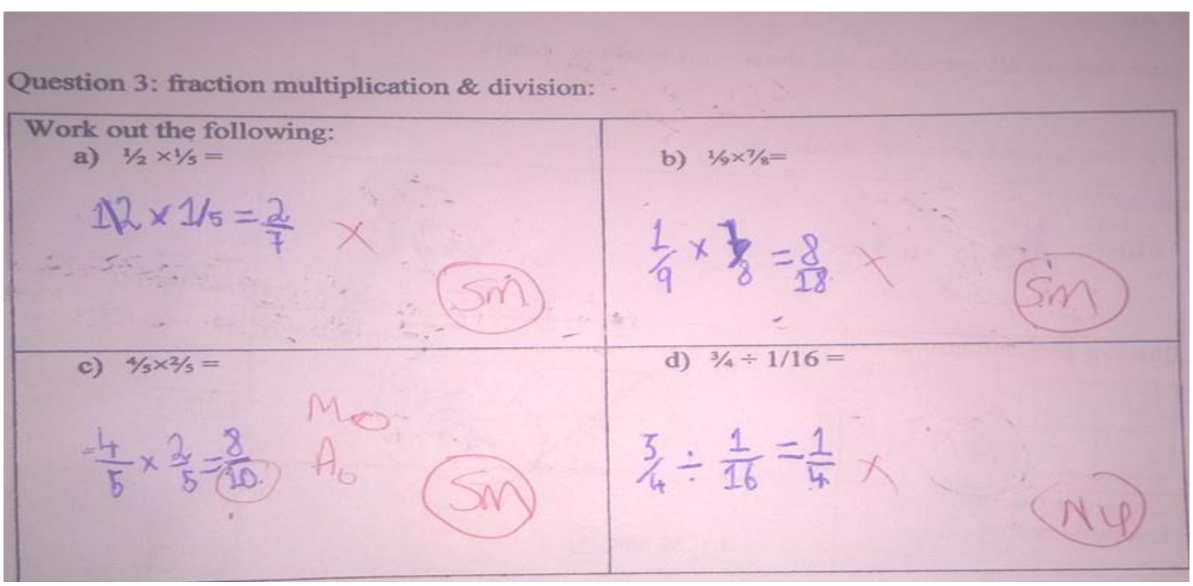

Figure 3: A sample of a learners who carried out addition instead of multiplication

In part (b), proportional distribution was as follows: PC-5(10%), CE-25(52%), PE-6(13%) and the rest had no understanding and those who did not write anything. The reasons for these percentages are similar for part (a), showing computational weakness among the learners since some of them, for example, worked out $^1/_9 \times ^7/_8 = ^7/_{81}$ instead of $^7/_{72}$. Likewise in part (c) the results showed in the table above are due to the similar reasons that have been discussed. Part (d) required the learners to work out $^3/_4 \div ^1/_{16}$. All the learners were challenged by this question as learners were categorized under NU and NR. Lastly, part (e) required the learners to write down a rule for multiplying and dividing fractions to show understanding of the algorithm carried out. The results from the table show that all the learners failed to give the expected response, since 43(90%) showed no understanding and those who never attempted this part. Even those who had an idea were classified under PC-5(10%).

Table 6: Learners competencies, process skills in problem solving in fractions

Q4	*CONCEPTS*	*MC*	*PC*	*SM*	*CE*	*PE*	*NU*	*NR*
a)	Problem solving:	0	5	0	0	0	16	27
		0%	10%	0%	0%	0%	33%	56%
b)		1	0	0	0	0	25	22
		2%	0%	0%	0%	0%	52%	46%
c)		3	4	0	0	0	23	18
		6%	8%	0%	0%	0%	48%	38%

Key: *Mathematically Correct (SC), Partial Correct (PC), Specific Misconception (SM), Conceptual Error (CE), Procedural Error (PE), No Understanding (NU) and No Response (NR)*

Question 4 was challenging for most learners, results showed that learners are challenged by solving problems related to fractions. Part (a) distribution was as follows PC-5(10%) while NU-16(33%) and the rest could not even attempt the question. Similar trends were deduced from parts (b) and (c). The question required the learners to solve a given word problem. The question was divided into three parts: Part a) was asking the learners to find the fraction of boys in a class of 80 pupils and a fraction of $^3/_8$ girls. There were no learners who answered this question correctly and those learners who answered were classified under PC. In Part b), the learners were asked to calculate the number of girls. Only 1(2%) learner answered correctly, showing all the working. The others showed no understanding while one did not respond. Part (c) required the learners to calculate the number of boys in the class and MC-3(6%) answered correctly, while PC-4 (8%) working and the rest showed no understanding and not response.

DISCUSION OF FINDINGS

The result of the study show that learners do experience some deficiencies in terms of process skills, competencies in handling mathematical knowledge and understanding in the topic fractions. These causations of the challenges in solving problems related to fractions have been presented according structure of the test.

a) Learners understanding, shading and comparing sizes of fractions

The study identified that most learners have no concrete understanding of a fraction. Siegler (2010) states that, learners need to have a basic conceptual understanding of fractions before the procedural understanding; learners should be exposed to the meaning of fractions, and according to Cramer, Monson, Whitney and Leavitt (2010), using the part-whole construct is an effective starting point for building meaning of fractions. In shading of fractions, most of the learners did not struggle with shading the fraction ¾ in a shape such as a square that was divided into four equal parts. However, most learners struggled with shading ½ in a shape such as hexagon that is divided into six equal parts. The common incorrect answer was shading the numerator only which changed the meaning of the given fraction which was ½ to $^1/_6$. Cramer et al. (2010) explain that this is caused by a misconception that learners thought the numerator and denominator were separate values. Van de Walle (2010) also adds that learners do not view a fraction as a single number. This is also similar to the findings of Ndalichako (2013), in the literature, which showed that, learners, when operating fractions, tend to treat numerators and denominators as separate entities, and as a result, they experience difficulty in solving questions related to fractions. The part of comparing the fractions by selecting the larger fraction between two fractions also seemed to be a challenge to learners. Most learners were selecting the fraction with a larger denominator.

b) Addition and subtraction of fractions with the same and different denominators.

In this part of the test, the results showed that some learners are challenged by applying fraction operations of adding and subtracting. Some learners were challenged by the fact that they did not have the conceptual understanding of the Lowest Common Multiple (LCM) method and therefore, they could not apply it. According to Arslan (2010), learning of fractions require both conceptual and procedural understanding.

Specific misconceptions were identified from this question, for example, some were operating on the fractions in terms of whole numbers. These learners, according to McNamara and Shaughnessy (2010), over-generalized the whole-number knowledge. Some of the learners were able to apply the LCM method to fractions with the same denominator but failed with fractions with different denominators. Those who were able were committing procedural errors whereby they would write a wrong LCM value or they would write the correct LCM value but work out and proceed wrongly. The question which required learners to write down a rule for adding and subtracting fractions was also a challenge for most learners. Arslan (2010) explains that most learners learn fractions procedurally, that is, they only memorize algorithm.

c) Multiplication and division of fractions with the same and those with different denominators.

This part of the test also revealed that learners are challenged by fraction operations, and in this case; multiplication and division. Some learners were committing conceptual errors, procedural errors, specific misconceptions and no understanding. Some learners were using the LCM method when working out fraction multiplication, that is, they were trying to maintain common denominators. According to Siegler et al. (2011), most learners are confused by why and when common denominators are maintained, leading to errors such as $^2/_5 \times {}^3/_5 = {}^6/_5$. This shows lack of understanding of the conceptual basis of fraction arithmetic procedures. Some learners in this question were challenged by the fact that they have not yet mastered their multiplication tables, and for that reasons they would make a lot of errors such as $^1/_9 \times {}^7/_8 = {}^7/_{81}$ instead of $^7/_{72}$. Moreover, the learners in this question were also challenged by fraction division. The common problem was failure to invert the second fraction so that they can multiply to get the answer. Mick and Kolb (2002) in Yim (2010) explain that learners need to understand the concept of division as the inverse of multiplication first, so that they can understand better the meaning of fraction division algorithms.

d) Problem solving

This part of the question was poorly performed by almost all the learners. This question required both conceptual and procedural understanding of fraction operations. According to Siegler (2010), learners need to understand underlying concepts before they can begin with the procedures. The problem-solving question, required the learners to first understand that the total number of pupils in the class make one whole of $^{8}/_{8}$. To find the fraction of the boys in the class, given that the fraction of the girls is $^{3}/_{8}$, the learners had to subtract $^{3}/_{8}$ from $^{8}/_{8}$ demonstrating that part-whole concept is understood. The other parts of the question required the learner to apply multiplication or division operations of fractions as well as subtraction. All these operations required conceptual and procedural understanding.

CONCLUSIONS

The study revealed the challenges faced by primary learners in solving problems related to fractions. The study also found that procedural and conceptual understanding is vital in fraction concepts. These are the results that were deduced from this study:

- Learners were able to carry out mathematical procedures without conceptual understanding of fractions in general, where learners had difficulties in separating whole number concepts from fraction concepts.
- Limited conceptual understanding of addition and subtraction of fraction where learners are required to equalize the denominators first before adding or subtracting.
- In Multiplication and division learners were committing conceptual errors, procedural errors, specific misconceptions and even limited understanding which showed partial understanding of both conceptual basis of fraction and arithmetic procedures.
- In problem solving, learners lacked a higher order in the application of fractions; they showed no understanding of the conceptual and procedural application of fractions.

Recommendations.

On the basis of the findings of this study, the following recommendations are suggested to be considered. They include:

- Developing practices and lessons that begin by exposing learners' perspectives about fractions and consistently exploring fractions such as one-half, one-fourth and one-tenth.
- Increasing the precision of the fraction language so that greater understanding of fraction units, fraction relationships and fraction operations can be promoted.
- Teachers should help learners to make real-life connections with fractions so that they can conceptualize how fractions operation works.
- Learners should be thoroughly taught the basic mathematical operations so that they can use them correctly. This should also include them being taught mathematical tables frequently, multiples, factors,

the meaning of the four basic operations and related mathematical operations before moving on to the fraction operations.

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